

Nonsingular Increasing Gravitational Potential for the Brane in 6D

Merab Gogberashvili^a and Douglas Singleton^b

^a Andronikashvili Institute of Physics, 6 Tamarashvili Str.
Tbilisi 0177, Georgia
E-mail: gogber@hotmail.com

^b Physics Dept., CSU Fresno, 2345 East San Ramon Ave. M/S 37
Fresno, CA 93740-8031, USA
E-mail: dougs@csufresno.edu

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Abstract

We present a new (1+3)-brane solution to Einstein equations in (1+5)-space. As distinct from previous models this solution is free of singularities in the full 6-dimensional space-time. The gravitational potential transverse to the brane is an increasing (but not exponentially) function and asymptotically approaches a finite value. The solution localizes the zero modes of all kinds of matter fields and Newtonian gravity on the brane. An essential feature of the model is that different kind of matter fields have different localization radii.

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The scenario where our world is associated with a brane embedded in a higher dimensional space-time with non-factorizable geometry has attracted a lot of interest since the appearance of papers [1, 2, 3]. In this model gravitons, which are allowed to propagate in the bulk, are confined on the brane because of a warped geometry. However, there are difficulties with the choice of a natural trapping mechanism for some matter fields. For example, in the existing (1+4)-dimensional models spin 0 and spin 2 fields are localized on the brane with an exponentially decreasing gravitational warp factor, spin 1/2 field are localized on the brane with an increasing factor [4], and spin 1 fields are not localized at all [5]. For the case of (1+5)-dimensions it was found that spin 0, spin 1 and spin 2 fields are localized on the brane with a decreasing warp factor and spin 1/2 fields again are localized with an increasing factor [6].

The reason why there are problems with localization of fermions in warped geometries is that in the Lagrangians for the fermions there appears an increasing exponential, coming from the metric tensors with upper indices, g^{AB} , and from the tetrads, h_B^A . As a result the action integral over the extra coordinates diverges, which is the signal for the non-localization of the fermionic fields. In both (1+4)-, or (1+5)-space models with warped geometry one is required to introduce some non-gravitational interaction in order to localize all the Standard Model particles.

For reasons of economy and to avoid charge universality obstruction [7] one would like to have a universal gravitational trapping mechanism for all fields. In our previous papers we found such a solution of the 6-dimensional Einstein equations in (2+4)- and (1+5)-spaces, which localized all kind of bulk fields on the brane [8, 9]. These solutions contain non-exponential scale factors, which increase from the brane, and asymptotically approach a finite value at infinity. In the paper [10] the solution of [9] was generalized to the case of n dimensions. In this paper we present a new solution to the Einstein equations in (1+5)-space, which, similar to the models [8, 9], also localizes all kind of physical fields on the brane, but is free of singularities in the full space-time. Because of this feature of the solution by setting realistic boundary conditions we are able to fix the free parameters of the model.

The general form of action of the gravitating system in six dimensions is

$$S = \int d^6x \sqrt{-^6g} \left[\frac{M^4}{2} (^6R + 2\Lambda) + ^6L \right] , \quad (1)$$

where $\sqrt{-^6g}$ is the determinant, M is the fundamental scale, 6R is the scalar curvature, Λ is the cosmological constant and 6L is the Lagrangian of matter fields. All of these quantities are six dimensional.

The 6-dimensional Einstein equations with stress-energy tensor T_{AB} are

$$^6R_{AB} - \frac{1}{2} g_{AB} ^6R = \frac{1}{M^4} (\Lambda g_{AB} + T_{AB}) . \quad (2)$$

Capital Latin indices run over $A, B, \dots = 0, 1, 2, 3, 5, 6$.

As in the papers [8, 9] for the metric of the 6-dimensional space-time we choose the ansatz

$$ds^2 = \phi^2(r) \eta_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta - \lambda(r) (dr^2 + r^2 d\theta^2) , \quad (3)$$

where the Greek indices $\alpha, \beta, \dots = 0, 1, 2, 3$ refer to 4-dimensional coordinates. The metric of ordinary 4-space, $\eta_{\alpha\beta}(x^\nu)$, has the signature $(+, -, -, -)$. The functions $\phi(r)$ and $\lambda(r)$ depend only on the extra radial coordinate, r , and thus are cylindrically symmetric in the transverse polar coordinates ($0 \leq r < \infty$, $0 \leq \theta < 2\pi$).

The ansatz (3) is different from the metric investigated in (1+5)-space brane models with warped geometry [6, 11, 12]

$$ds^2 = \phi^2(r) \eta_{\alpha\beta}(x^\nu) dx^\alpha dx^\beta - dr^2 - \lambda(r) d\theta^2 . \quad (4)$$

In (3) the independent metric function of the extra space, $\lambda(r)$, serves as a conformal factor for the Euclidean 2-dimensional metric of the transverse space, just as the function $\phi^2(r)$ does for the 4-dimensional part. However, in (4) the function $\lambda(r)$ multiplies only the angular part of the metric and corresponds to a cone-like geometry of a string-like defect with a singularity on the brane at $r = 0$. Only in the trivial case $\lambda = r^2$ is the metric (4) regular on the brane.

The stress-energy tensor T_{AB} is assumed to have the form

$$T_{\mu\nu} = -g_{\mu\nu} F(r), \quad T_{ij} = -g_{ij} K(r), \quad T_{i\mu} = 0 . \quad (5)$$

Using the ansatz (3), the energy-momentum conservation equation

$$\nabla^A T_{AB} = \frac{1}{\sqrt{-^6g}} \partial_A (\sqrt{-^6g} T^{AB}) + \Gamma_{CD}^B T^{CD} = 0 , \quad (6)$$

gives a relationship between the two source functions $F(r)$ and $K(r)$ from (5)

$$K' + 4 \frac{\phi'}{\phi} (K - F) = 0 . \quad (7)$$

We want to point out a problem associated with the source functions (5). In general the Einstein equations have an infinite number of solutions generated by different matter energy-momentum tensors, most of which have no clear physical meaning. There is a great freedom in the choice of $F(r)$ and $K(r)$; there are no other restrictions, except (7), on their form. It is not easy to construct realistic source functions from fundamental matter fields so that the brane is a stable, localized object. We shall determine the sources $F(r)$ and $K(r)$ from some general physical assumptions that they are smooth functions of the radial coordinate r , describe a continuous matter distribution for all r , and that they decrease outside the brane $r > \epsilon$, where ϵ is the brane width.

For a string-like defect, which corresponds to the metric ansatz (4), a set of n scalar functions with a Higgs potential (which breaks the global $SO(n)$ symmetry to $SO(n-1)$) are used as a source to make the brane stable. In this case topological arguments guarantee the stability of the brane because the $(n-1)$ homotopy group is the integers: $\Pi_{n-1}(SO(n-1)) = \mathbb{Z}$ (see, for example [6]).

In our case we require the condition of classical stability [13], that the total momentum of the brane-matter configuration in the direction of the extra dimensions is zero

$$P_i = \int t_i^A dS_A = 0 , \quad (8)$$

where t_A^B is the total energy-momentum pseudo-tensor of gravitation plus matter fields on the brane. For t_A^B one can choose, for example, the so-called Lorenz energy-momentum complex

$$t_A^B = \frac{1}{\sqrt{-6}g} \partial_C \left[\sqrt{-6}g [g^{BD} g^{CE} (\partial_D g_{AE} - \partial_E g_{AD})] \right] . \quad (9)$$

To obey the stability condition (8), brane solutions must satisfy the conditions

$$t_i^\alpha = t_i^j = 0 . \quad (10)$$

Using (9) from (10) we obtain that the metric tensor of the transverse space is a function of r only and that $g_{iA} = 0$. These conditions are satisfied by both ansätze (3) and (4). One can have a condition similar to (10) separately on the gravitational energy-momentum pseudo-tensor, which gives [13]

$$\partial_i \Gamma_{\beta\gamma}^\alpha = 0 , \quad (11)$$

where $\Gamma_{\beta\gamma}^\alpha$ are components of the 6-dimensional Christoffel symbols. From (11) follows the standard form of the 4-dimensional part of the brane metric tensor, $g_{\alpha\beta} = \phi^2(r) \eta_{\alpha\beta}(x^\nu)$. For the case of more than six dimensions θ dependent coefficients will appear in the angular part of the metrics (3) and (4) and the condition of classical stability (8) fails. This may be one of the reasons why brane solutions in more than six dimensions are singular [14].

The condition of classical stability (8) is satisfied for both metrics (3) and (4). It is expected that, as for the string-like defect (4), one can find for (3) particle physics models for the source (5) corresponding to stable brane configurations.

To solve equations (2) we require that the 4-dimensional Einstein equations have the ordinary form without a cosmological term

$$R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R = 0 . \quad (12)$$

The Ricci tensor in four dimensions $R_{\alpha\beta}$ is constructed from the 4-dimensional metric tensor $\eta_{\alpha\beta}(x^\nu)$ in the standard way. Then with the ansätze (3) and (5) the Einstein field equations (2) become

$$\begin{aligned} 3 \frac{\phi''}{\phi} + 3 \frac{\phi'}{r\phi} + 3 \frac{(\phi')^2}{\phi^2} + \frac{1}{2} \frac{\lambda''}{\lambda} - \frac{1}{2} \frac{(\lambda')^2}{\lambda^2} + \frac{1}{2} \frac{\lambda'}{r\lambda} &= \frac{\lambda}{M^4} [F(r) - \Lambda] , \\ \frac{\phi'\lambda'}{\phi\lambda} + 2 \frac{\phi'}{r\phi} + 3 \frac{(\phi')^2}{\phi^2} &= \frac{\lambda}{2M^4} [K(r) - \Lambda] , \\ 2 \frac{\phi''}{\phi} - \frac{\phi'\lambda'}{\phi\lambda} + 3 \frac{(\phi')^2}{\phi^2} &= \frac{\lambda}{2M^4} [K(r) - \Lambda] , \end{aligned} \quad (13)$$

where the prime $= \partial/\partial r$. These equations are for the $\alpha\alpha$, rr , and $\theta\theta$ components respectively.

Subtracting the rr from the $\theta\theta$ equation and multiplying by ϕ/ϕ' we arrive at

$$\frac{\phi''}{\phi'} - \frac{\lambda'}{\lambda} - \frac{1}{r} = 0 . \quad (14)$$

This equation has the solution

$$\lambda(r) = \frac{\rho^2 \phi'}{r} , \quad (15)$$

where ρ is an integration constant with units of length.

System (13), after the insertion of (15), reduces to only one independent equation. Taking either the rr , or $\theta\theta$ component of these equations and multiplying it by $r\phi^4$ gives

$$r\phi^3\phi'' + \phi^3\phi' + 3r\phi^2(\phi')^2 = \frac{\rho^2\phi^4\phi'}{2M^4} [K(r) - \Lambda] . \quad (16)$$

In our previous paper [9] the source functions $F(r)$ and $K(r)$ outside the core $r > \epsilon$ were taken to have the form

$$F(r > \epsilon) = K(r > \epsilon) = \frac{f}{\phi^2}, \quad (17)$$

where f is some constant. Then from the Einstein equations the following solution was found

$$\phi = a \frac{r^b - c^b}{r^b + c^b}, \quad (18)$$

where

$$a = \sqrt{\frac{5f}{3\Lambda}}, \quad b = \frac{a\Lambda\rho^2}{5M^4}, \quad c^b = \epsilon^b \frac{a-1}{a+1} \quad (19)$$

are integration constants. The solution (18) is an increasing function from the brane to some finite value at infinity

$$\phi(\infty) = a = \sqrt{\frac{5f}{3\Lambda}} > 1. \quad (20)$$

The factor $1/\phi^2(r)$ has δ -like behavior outside the core and the source functions (17) decrease as required.

In [9] it was shown that the solution (18) provides a universal, gravitational trapping for all kinds of matter fields. However, in this model we did not specify source functions on the brane $0 \leq r \leq \epsilon$ and there were a large number of free parameters. Now we want to choose the source functions $F(r)$ and $K(r)$ everywhere, so that the solution ϕ will localize all kind of physical fields on the brane and be a regular function in the full 6-dimensional space-time.

We require for ϕ the following boundary conditions near the origin $r = 0$

$$\phi(r \rightarrow 0) \approx 1 + dr^2, \quad \phi'(r \rightarrow 0) \approx 2dr, \quad (21)$$

where d is some constant. At infinity we want $\phi(r)$ to behave as

$$\phi(r \rightarrow \infty) \rightarrow a, \quad \phi'(r \rightarrow \infty) \rightarrow 0, \quad (22)$$

where $a > 1$ is some constant. Since the function ϕ' is proportional to the metric of the extra 2-space, the boundary conditions (22) imply that at infinity $\lambda \rightarrow 0$ and the effective geometry is 4-dimensional.

The source functions $F(r)$ and $K(r)$, which satisfy restriction (7) and give a desirable solution were found recently in the paper [15]

$$F(r) = \frac{f_1}{2\phi^2} + \frac{3f_2}{4\phi}, \quad K(r) = \frac{f_1}{\phi^2} + \frac{f_2}{\phi}, \quad (23)$$

where f_1, f_2 are constants. Note that these source functions do not have a vanishing value at $r \rightarrow \infty$, due to the asymptotic behavior of ϕ given in (22).

Substituting (23) into (16), taking its first integral and setting the integration constant to zero yields [8, 15]

$$r\phi' = \frac{\rho^2\Lambda}{10M^4} \left(\frac{5f_1}{3\Lambda} + \frac{5f_2}{4\Lambda}\phi - \phi^2 \right). \quad (24)$$

By introducing the parameters A and a such that

$$\frac{\rho^2\Lambda}{10M^4} = A, \quad f_1 = -\frac{3\Lambda}{5}a, \quad f_2 = \frac{4\Lambda}{5}(a+1), \quad (25)$$

equation (24) becomes

$$r\phi' = A[-a + (a+1)\phi - \phi^2]. \quad (26)$$

Equation (26) is easy to integrate [15]

$$\phi = \frac{c^b + ar^b}{c^b + r^b}, \quad (27)$$

where $b = A(a - 1)$ and c are integration constants. From the boundary conditions (21) it follows

$$b = A(a - 1) = 2 . \quad (28)$$

The width of the brane ϵ corresponds to the inflection point of the function ϕ (where the second derivative of ϕ become zero and then changes sign). From the condition $\phi''(r = \epsilon) = 0$ we can fix the integration constant c in (27)

$$c^2 = 3\epsilon^2 . \quad (29)$$

Finally the solution ϕ corresponding to a non-singular transverse gravitational potential for the brane has the form

$$\phi = \frac{3\epsilon^2 + ar^2}{3\epsilon^2 + r^2} . \quad (30)$$

From the condition that we have a 6-dimensional Minkowski metric on the brane, $\lambda(r = 0) = 1$, (any other value corresponds only to a re-scaling of the extra coordinates) we can fix also the integration constant in (15)

$$\rho^2 = \frac{3\epsilon^2}{2(a - 1)} . \quad (31)$$

Then using (28) the brane width can be expressed in terms of the bulk cosmological constant and fundamental scale

$$\epsilon^2 = \frac{40M^4}{3\Lambda} . \quad (32)$$

Now the metric tensor of the transverse space (15) is not dependent on a and has the form

$$\lambda = \frac{9\epsilon^4}{(3\epsilon^2 + r^2)^2} . \quad (33)$$

Using solutions (27), (33) and the relationship (15) to integrate the gravitational part of the action integral (1) over the extra coordinates we find

$$\begin{aligned} S_g &= \frac{M^4}{2} \int dx^6 \sqrt{-^6g} {}^6R = \frac{M^4}{2} \int_0^{2\pi} d\theta \int_0^\infty dr r \phi^2 \lambda \int dx^4 \sqrt{-\eta} R \\ &= \rho^2 \pi M^4 \int_1^a d\phi \phi^2 \int dx^4 \sqrt{-\eta} R = \frac{M^4}{2} \epsilon^2 \pi (a^2 + a + 1) \int dx^4 \sqrt{-\eta} R , \end{aligned} \quad (34)$$

where R and η are respectively the scalar curvature and determinant, in four dimensions.

The formula for the effective Planck scale in our model, which is two times the numerical factor in front of the last integral in (34)

$$m_{Pl}^2 = M^4 \pi \epsilon^2 (a^2 + a + 1) , \quad (35)$$

is similar to those from the “large” extra dimensions model [1]. The differences are, the presence of the value of gravitational potential at extra infinity, a , in (35), and that the radius of the extra dimensions is replaced by the brane width ϵ , which, as seen from (32), is expressed by the ratio of the fundamental scale M and the cosmological constant Λ .

The normalization condition for a physical field, that its action integral over the extra coordinates r, θ converges, is also the condition for its localization. As was shown in [9] Newtonian gravity is localized on the brane, since the action integral for gravity, (34), is convergent over the extra space. However, the wave-functions of a localized matter field can be spread out from the brane more widely then the brane width ϵ . In order not to have contradictions with experimental facts, such as charge conservation [7], the parameters of the model must be chosen in a proper way.

When wave-functions of matter fields in six dimensions are peaked near the brane in the transverse dimensions there wave-functions on the brane can be factorized as

$$\Xi(x^A) = \frac{\xi(x^\nu)}{\kappa} , \quad (36)$$

where the parameter κ is the value of the constant zero mode with the dimension of length. These parameters can be found from the normalization condition for zero modes

$$\int_0^{2\pi} d\theta \int_0^\infty dr \sqrt{-6g} \frac{1}{\kappa^2} = \sqrt{-\eta} , \quad (37)$$

which also guarantees the validity of the equivalence principle for different kinds of particles.

Let us consider the situation with the localization of particular matter fields. If we assume that the zero mode of a spin-0 field, Φ , in six dimensions is independent of the extra coordinates its action can be brought to the form [9]

$$\begin{aligned} S_\Phi &= \int d^6x \sqrt{-6g} {}^6L_\Phi(x^A) = \frac{2\pi}{\kappa_\Phi^2} \int_0^\infty dr r \phi^2 \lambda \int d^4x \sqrt{-\eta} L_\Phi(x^\nu) = \\ &= \frac{2\pi\rho^2}{\kappa_\Phi^2} \int_1^a \phi^2 d\phi \int d^4x \sqrt{-\eta} L_\Phi(x^\nu) = \frac{\epsilon^2\pi(a^2 + a + 1)}{\kappa_\Phi^2} \int d^4x \sqrt{-\eta} L_\Phi(x^\nu) , \end{aligned} \quad (38)$$

where $L_\Phi(x^\nu)$ is the ordinary 4-dimensional Lagrangian of the spin-0 field and κ_Φ is value of the constant zero mode. The integral over r, θ in (38) is finite and the spin-0 field is localized on the brane.

The action for a vector field in the case of constant extra components ($A_i = \text{const}$) also reduces to the 4-dimensional Yang-Mills action multiplied an integral over the extra coordinates [9]

$$\begin{aligned} S_A &= \int d^6x \sqrt{-6g} {}^6L_A(x^B) = \frac{2\pi}{\kappa_A^2} \int_0^\infty dr r \lambda \int d^4x \sqrt{-\eta} L_A(x^\nu) \\ &= \frac{2\pi\rho^2}{\kappa_A^2} \int_1^a d\phi \int d^4x \sqrt{-\eta} L_A(x^\nu) = \frac{3\epsilon^2\pi}{\kappa_A^2} \int d^4x \sqrt{-\eta} L_A(x^\nu) , \end{aligned} \quad (39)$$

where κ_A is the value of the zero mode of the vector field. The extra integral in (39) is also finite and the gauge field is localized on the brane.

The factorization of the zero mode of a 6-dimensional spinor field in the ansatz (3) is different from the definition (36), having instead the form [9]

$$\Psi(x^A) = \frac{\psi(x^\nu)}{\kappa_\Psi \phi^2 (r\phi')^{1/4}} , \quad (40)$$

where κ_Ψ is the value of the constant zero mode. Integrating the 6-dimensional action of fermions over the extra coordinates, using the explicit form (30), yields

$$\begin{aligned} S_\Psi &= \int d^6x \sqrt{-6g} {}^6L_\Psi(x^A) = \frac{2\pi\rho^2}{\kappa_\Psi^2} \int_0^\infty dr \sqrt{\frac{\phi'}{r\phi^2}} \int d^4x \sqrt{-\eta} L_\Psi(x^\nu) \\ &= \frac{3\pi^2\epsilon^2}{\kappa_\Psi^2 \sqrt{2a(a-1)}} \int d^4x \sqrt{-\eta} L_\Psi(x^\nu) , \end{aligned} \quad (41)$$

where L_Ψ is the 4-dimensional Dirac Lagrangian. The extra $1/\phi$ dependence in the second integral of (41) comes from the tetrad functions with upper index in the definition of the Dirac gamma matrices for the ansatz (3). The integral in (41) over r and θ is finite and Dirac fermions are localized on the brane.

Equating the coefficients of action integrals (38), (39) and (41) to 1, so as to satisfy the normalization condition (37), and to guarantee the equivalence principle for gravity, we find the values of the zero modes for spin 0, spin 1 and spin 1/2 fields

$$\kappa_\Phi^2 = \pi\epsilon^2(a^2 + a + 1) , \quad \kappa_A^2 = 3\pi\epsilon^2 , \quad \kappa_\Psi^2 = \frac{3\pi^2\epsilon^2}{\sqrt{2a(a-1)}} , \quad (42)$$

which are used to parameterize the 4-dimensional fields in the Lagrangians.

Within our model we now want to find the the positions where the zero modes are localized as well as the localization radii of the different fields.

From (38) the effective zero mode wave-function of the scalar field in flat space can be defined as

$$\Phi_0(r) = \sqrt{\frac{2\pi r \phi^2 \lambda}{\kappa_\Phi^2}} = \frac{\sqrt{2\pi} 3\epsilon^2}{\kappa_\Phi} \sqrt{r} \frac{(3\epsilon^2 + ar^2)}{(3\epsilon^2 + r^2)^2}, \quad (43)$$

where κ_Φ has the value (42). Function (43) is zero at infinity ($r \rightarrow \infty$) and on the brane ($r = 0$) and has a maximal value at some localization distance, d_Φ , between the brane ($r = 0$) and infinity ($r = \infty$). This localization distance, d_Φ , and the localization radius r_Φ , can be found by equating the first and second derivatives (to find the maximum and inflection point of the function) of (43) to zero respectively. For the localization distance this yields

$$d_\Phi^2 = \epsilon^2 \frac{5a - 7 + \sqrt{49 - 58a + 25a^2}}{2a}. \quad (44)$$

From this formula we see that since $a > 1$ the maximum of the wave-function of scalar fields (43) is located outside the brane $d_\Phi > \epsilon$.

Setting the second derivative of (43) to zero gives

$$5ar^6 + (63 - 66a)\epsilon^2 r^4 - (102 - 45a)\epsilon^4 r^2 - 9\epsilon^6 = 0. \quad (45)$$

This is an effectively cubic equation for r^2 , which has one real and two complex solutions. The radius, r_Φ , of the zero mode scalar wave-function is given by the real solution to (45). The expressions of the solutions of (45), which can be obtained using a symbolic mathematics program such as *Mathematica*, are extremely long and we do not write them here explicitly.

From (39) the effective wave-function of the vector field zero mode takes the form

$$A_0(r) = \sqrt{\frac{2\pi r \lambda}{\kappa_A^2}} = \frac{\sqrt{2\pi} 3\epsilon^2}{\kappa_A} \frac{\sqrt{r}}{3\epsilon^2 + r^2}. \quad (46)$$

This function is also zero on the brane and at the infinity, and has a maximal value at some distance, d_A , in between. Again setting the first and second derivatives of (46) to zero we find

$$d_A = \epsilon, \quad r_A = \sqrt{\frac{\epsilon^2(9 + 4\sqrt{6})}{5}} \approx 1.9\epsilon. \quad (47)$$

So the peaks of the vector wave-functions are located exactly at the edge of the brane, $r = \epsilon$ and the radius of localization is approximately 2ϵ .

For the effective fermionic zero modes from (41) we have

$$\psi_0(r) = \sqrt{\frac{2\pi \rho^2}{\kappa_\Psi^2} \left(\frac{\phi'}{r\phi^2} \right)^{1/2}} = \left[\frac{54\pi^2 \epsilon^6}{\kappa_\Psi^4 (a - 1)} \right]^{1/4} \frac{1}{\sqrt{3\epsilon^2 + ar^2}}. \quad (48)$$

This function is zero at infinity, but unlike the wave-functions of the scalar and vector zero modes, the peak of the fermion wave-function coincides with brane location, $r = 0$. From the inflection point of the function (48), which is found by equating the second derivative of (48) to zero, we obtain the localization radius for fermions

$$r_\Psi = \epsilon \sqrt{\frac{3}{2a}}. \quad (49)$$

From this formula we see that since $a > 1$ the peaks of fermionic wave-functions on the brane are very sharp.

After introducing interactions between the various fields one might be able to use the various overlappings of the wave functions of the spin 0, spin 1 and spin 1/2 fields in speculations to explain the different types of mass hierarchies in particle physics. For example, in the split fermion model [16] localization of different species of fermions (not fields with different spins as in our case) at different points of a single thick brane was used to solve the hierarchy problem.

To summarize, in this paper it is shown that for a realistic form of the brane stress-energy, there exists a static, non-singular solution of the 6-dimensional Einstein equations, which provides a gravitational trapping of 4-dimensional gravity and matter fields on the brane. An essential feature of the model is that different kinds of matter fields have different localization distances from the brane. This property is in principle experimentally testable.

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References

- [1] N. Arkani-Hamed, S. Dimopoulos and G.Dvali, Phys. Lett., **B429**, 263 (1998);
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett., **B436**, 257 (1998).
- [2] M. Gogberashvili, Int. J. Mod. Phys., **D 11**, 1635 (2002); *ibid.* **D 11**, 1639 (2002).
- [3] L. Randall and R. Sundrum, Phys. Rev. Lett., **83**, 3370 (1999); *ibid.* **83**, 4690 (1999).
- [4] B. Bajc and G. Gabadadze, Phys. Lett., **B 474**, 282 (2000).
- [5] A. Pomarol, Phys. Lett., **B 486**, 153 (2000).
- [6] I. Oda, Phys. Rev., **D 62**, 126009 (2000).
- [7] S.L. Dubovsky, V.A. Rubakov and P.G. Tinyakov, JHEP, **0008**, 041 (2000).
- [8] M. Gogberashvili and P. Midodashvili, Phys. Lett., **B 515**, 447 (2001); Europhys. Lett., **61**, 308 (2003).
- [9] M. Gogberashvili and D. Singleton, hep-th/0305241.
- [10] I. Oda, Phys. Lett., **B 571**, 235 (2003).
- [11] R. Gregory, Phys. Rev. Lett., **84**, 2564 (2000).
- [12] T. Gherghetta and M. Shaposhnikov, Phys. Rev. Lett., **85**, 240 (2000).
- [13] M. Gogberashvili, Mod. Phys. Lett., **14**, 2025 (1999).
- [14] T. Gherghetta, E. Roessl and M. Shaposhnikov, Phys. Lett., **B 491**, 353 (2000).
- [15] P. Midodashvili, hep-th/0308051.
- [16] N. Arkani-Hamed and M. Schmaltz, Phys. Rev., **D 61**, 033005 (2000);
S.J. Huber, Nucl. Phys., **B 666**, 269 (2003);
Y. Grossman and G. Perez, Phys. Rev., **D 67**, 015011 (2003).